### 7.4.1 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 7 materials.

In Exercises 1 - 8, graph the ellipse. Find the center, the lines which contain the major and minor axes, the vertices, the endpoints of the minor axis, the foci and the eccentricity.

For help with these exercises click on the resource below:

- Graphing ellipses

1. $\frac{x^2}{169} + \frac{y^2}{25} = 1$
2. $\frac{x^2}{9} + \frac{y^2}{25} = 1$
3. $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$
4. $\frac{(x+5)^2}{16} + \frac{(y-4)^2}{1} = 1$
5. $\frac{(x-1)^2}{10} + \frac{(y-3)^2}{11} = 1$
6. $\frac{(x-1)^2}{9} + \frac{(y+3)^2}{4} = 1$
7. $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{20} = 1$
8. $\frac{(x-4)^2}{8} + \frac{(y-2)^2}{18} = 1$

In Exercises 9 - 14, put the equation in standard form. Find the center, the lines which contain the major and minor axes, the vertices, the endpoints of the minor axis, the foci and the eccentricity.

For help with these exercises click on one of the resources below:

- Completing the square to put the equation of the ellipse in standard form
- Graphing ellipses

9. $9x^2 + 25y^2 - 54x - 50y - 119 = 0$
10. $12x^2 + 3y^2 - 30y + 39 = 0$
11. $5x^2 + 18y^2 - 30x + 72y + 27 = 0$
12. $x^2 - 2x + 2y^2 - 12y + 3 = 0$
13. $9x^2 + 4y^2 - 4y - 8 = 0$
14. $6x^2 + 5y^2 - 24x + 20y + 14 = 0$

In Exercises 15 - 20, find the standard form of the equation of the ellipse which has the given properties.

For help with these exercises click on the resource below:

- Finding the equation of an ellipse

15. Center $(3, 7)$, Vertex $(3, 2)$, Focus $(3, 3)$
16. Foci $(0, \pm 5)$, Vertices $(0, \pm 8)$.
17. Foci $(\pm 3, 0)$, length of the Minor Axis 10

18. Vertices $(3, 2)$, $(13, 2)$; Endpoints of the Minor Axis $(8, 4)$, $(8, 0)$
19. Center $(5, 2)$, Vertex $(0, 2)$, eccentricity $\frac{1}{2}$
20. All points on the ellipse are in Quadrant IV except $(0, -9)$ and $(8, 0)$. (One might also say that the ellipse is "tangent to the axes" at those two points.)
21. Repeat Example 7.4.5 for a whispering gallery 200 feet wide and 75 feet tall.
22. An elliptical arch is constructed which is 6 feet wide at the base and 9 feet tall in the middle. Find the height of the arch exactly 1 foot in from the base of the arch. Compare your result with your answer to Exercise 21 in Section 7.3.
23. The Earth's orbit around the sun is an ellipse with the sun at one focus and eccentricity $e \approx 0.0167$. The length of the semimajor axis (that is, half of the major axis) is defined to be 1 astronomical unit (AU). The vertices of the elliptical orbit are given special names: 'aphelion' is the vertex farthest from the sun, and 'perihelion' is the vertex closest to the sun. Find the distance in AU between the sun and aphelion and the distance in AU between the sun and perihelion.
24. The graph of an ellipse clearly fails the Vertical Line Test, Theorem 1.1, so the equation of an ellipse does not define $y$ as a function of $x$. However, much like with circles and horizontal parabolas, we can split an ellipse into a top half and a bottom half, each of which would indeed represent $y$ as a function of $x$. With the help of your classmates, use your calculator to graph the ellipses given in Exercises 1 - 8 above. What difficulties arise when you plot them on the calculator?
25. Some famous examples of whispering galleries include St. Paul's Cathedral in London, England, National Statuary Hall in Washington, D.C., and The Cincinnati Museum Center. With the help of your classmates, research these whispering galleries. How does the whispering effect compare and contrast with the scenario in Example 7.4.5?
26. With the help of your classmates, research "extracorporeal shock-wave lithotripsy". It uses the reflective property of the ellipsoid to dissolve kidney stones.

**Checkpoint Quiz 7.4**

1. Put $4x^2 + y^2 - 6y + 5 = 0$ into standard form and graph. Find the center, vertices, and foci.

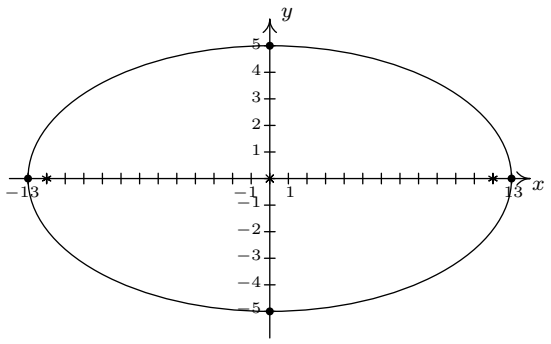
2. Find the equation of the ellipse with foci $(1, 0)$ and $(3, 0)$ which has a vertex at the origin.

For worked out solutions to this quiz, click the link below:

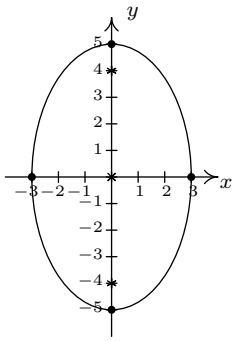
- Quiz Solution

### 7.4.2 Answers

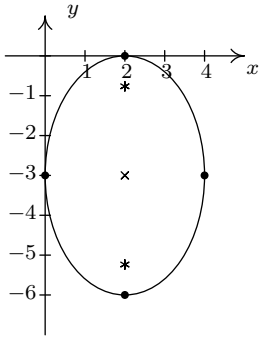
1. $\dfrac{x^2}{169} + \dfrac{y^2}{25} = 1$
   Center $(0, 0)$
   Major axis along $y = 0$
   Minor axis along $x = 0$
   Vertices $(13, 0)$, $(-13, 0)$
   Endpoints of Minor Axis $(0, -5)$, $(0, 5)$
   Foci $(12, 0)$, $(-12, 0)$
   $e = \frac{12}{13}$



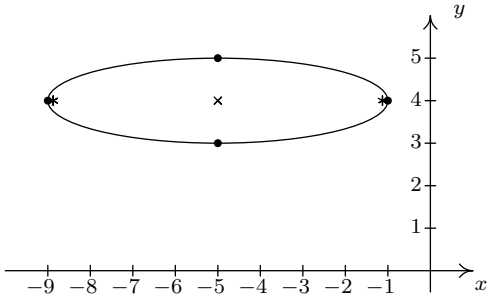

2. $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$
   Center $(0, 0)$
   Major axis along $x = 0$
   Minor axis along $y = 0$
   Vertices $(0, 5)$, $(0, -5)$
   Endpoints of Minor Axis $(-3, 0)$, $(3, 0)$
   Foci $(0, -4)$, $(0, 4)$
   $e = \frac{4}{5}$




3. $\dfrac{(x-2)^2}{4} + \dfrac{(y+3)^2}{9} = 1$
   Center $(2, -3)$
   Major axis along $x = 2$
   Minor axis along $y = -3$
   Vertices $(2, 0)$, $(2, -6)$
   Endpoints of Minor Axis $(0, -3)$, $(4, -3)$
   Foci $(2, -3 + \sqrt{5})$, $(2, -3 - \sqrt{5})$
   $e = \frac{\sqrt{5}}{3}$

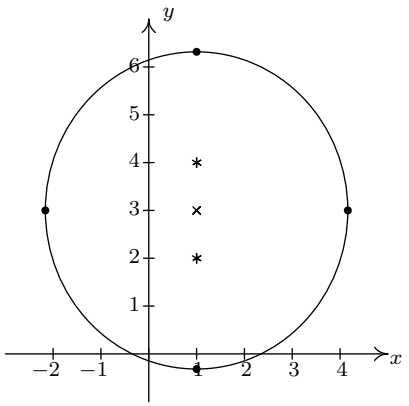



4. $\dfrac{(x+5)^2}{16} + \dfrac{(y-4)^2}{1} = 1$
   Center $(-5, 4)$
   Major axis along $y = 4$
   Minor axis along $x = -5$
   Vertices $(-9, 4)$, $(-1, 4)$
   Endpoints of Minor Axis $(-5, 3)$, $(-5, 5)$
   Foci $(-5 + \sqrt{15}, 4)$, $(-5 - \sqrt{15}, 4)$
   $e = \frac{\sqrt{15}}{4}$



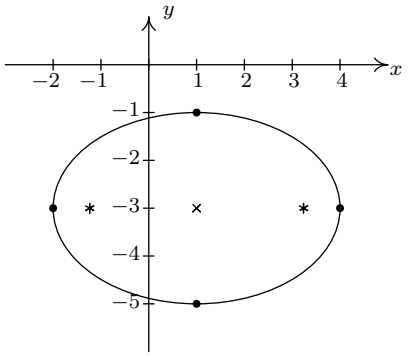
5. $\dfrac{(x-1)^2}{10} + \dfrac{(y-3)^2}{11} = 1$

Center $(1, 3)$
Major axis along $x = 1$
Minor axis along $y = 3$
Vertices $(1, 3+\sqrt{11})$, $(1, 3-\sqrt{11})$
Endpoints of the Minor Axis
$(1-\sqrt{10}, 3)$, $(1+\sqrt{10}, 3)$
Foci $(1, 2)$, $(1, 4)$
$e = \frac{\sqrt{11}}{11}$



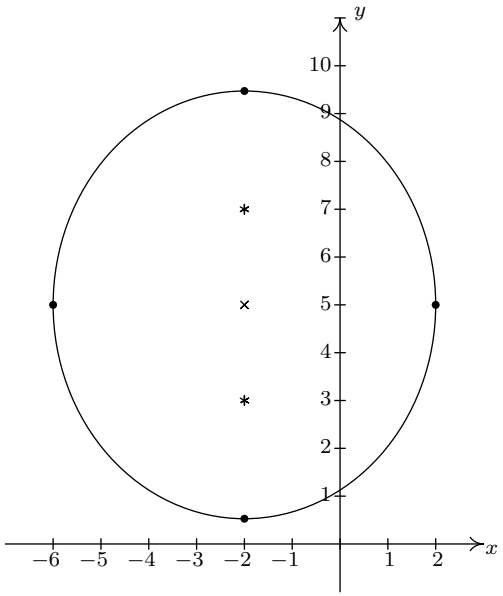

6. $\dfrac{(x-1)^2}{9} + \dfrac{(y+3)^2}{4} = 1$

Center $(1, -3)$
Major axis along $y = -3$
Minor axis along $x = 1$
Vertices $(4, -3)$, $(-2, -3)$
Endpoints of the Minor Axis $(1, -1)$, $(1, -5)$
Foci $(1+\sqrt{5}, -3)$, $(1-\sqrt{5}, -3)$
$e = \frac{\sqrt{5}}{3}$

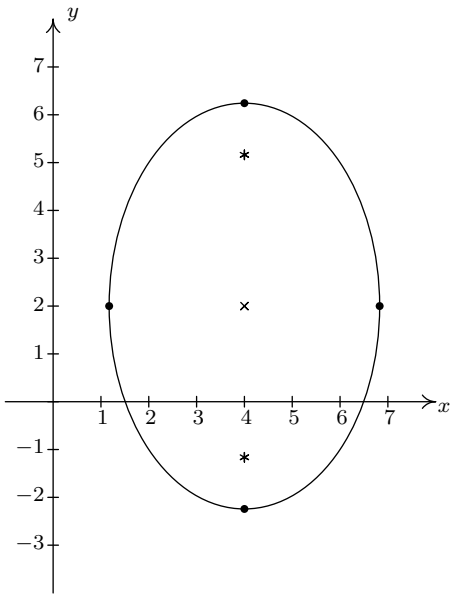



7. $\dfrac{(x+2)^2}{16} + \dfrac{(y-5)^2}{20} = 1$

Center $(-2, 5)$
Major axis along $x = -2$
Minor axis along $y = 5$
Vertices $(-2, 5+2\sqrt{5})$, $(-2, 5-2\sqrt{5})$
Endpoints of the Minor Axis $(-6, 5)$, $(2, 5)$
Foci $(-2, 7)$, $(-2, 3)$
$e = \frac{\sqrt{5}}{5}$



8. $\dfrac{(x-4)^2}{8}+\dfrac{(y-2)^2}{18}=1$
Center $(4,2)$
Major axis along $x=4$
Minor axis along $y=2$
Vertices $(4,2+3\sqrt{2})$, $(4,2-3\sqrt{2})$
Endpoints of the Minor Axis
$(4-2\sqrt{2},2)$, $(4+2\sqrt{2},2)$
Foci $(4,2+\sqrt{10})$, $(4,2-\sqrt{10})$
$e=\frac{\sqrt{5}}{3}$




9. $\dfrac{(x-3)^2}{25}+\dfrac{(y-1)^2}{9}=1$
Center $(3,1)$
Major Axis along $y=1$
Minor Axis along $x=3$
Vertices $(8,1)$, $(-2,1)$
Endpoints of Minor Axis $(3,4)$, $(3,-2)$
Foci $(7,1)$, $(-1,1)$
$e=\frac{4}{5}$

10. $\dfrac{x^2}{3}+\dfrac{(y-5)^2}{12}=1$
Center $(0,5)$
Major axis along $x=0$
Minor axis along $y=5$
Vertices $(0,5-2\sqrt{3}),(0,5+2\sqrt{3})$
Endpoints of Minor Axis $(-\sqrt{3},5)$, $(\sqrt{3},5)$
Foci $(0,2),(0,8)$
$e=\frac{\sqrt{3}}{2}$

11. $\dfrac{(x-3)^2}{18}+\dfrac{(y+2)^2}{5}=1$
Center $(3,-2)$
Major axis along $y=-2$
Minor axis along $x=3$
Vertices $(3-3\sqrt{2},-2),(3+3\sqrt{2},-2)$
Endpoints of Minor Axis $(3,-2+\sqrt{5})$, $(3,-2-\sqrt{5})$
Foci $(3-\sqrt{13},-2),(3+\sqrt{13},-2)$
$e=\frac{\sqrt{26}}{6}$

12. $\dfrac{(x-1)^2}{16}+\dfrac{(y-3)^2}{8}=1$
Center $(1,3)$
Major Axis along $y=3$
Minor Axis along $x=1$
Vertices $(5,3)$, $(-3,3)$
Endpoints of Minor Axis $(1,3+2\sqrt{2})$, $(1,3-2\sqrt{2})$
Foci $(1+2\sqrt{2},3)$, $(1-2\sqrt{2},3)$
$e=\frac{\sqrt{2}}{2}$

13. $\dfrac{x^2}{1} + \dfrac{4\left(y-\frac{1}{2}\right)^2}{9} = 1$
Center $\left(0, \frac{1}{2}\right)$
Major Axis along $x = 0$ (the $y$-axis)
Minor Axis along $y = \frac{1}{2}$
Vertices $(0, 2)$, $(0, -1)$
Endpoints of Minor Axis $\left(-1, \frac{1}{2}\right)$, $\left(1, \frac{1}{2}\right)$
Foci $\left(0, \frac{1+\sqrt{5}}{2}\right)$, $\left(0, \frac{1-\sqrt{5}}{2}\right)$
$e = \frac{\sqrt{5}}{3}$

14. $\dfrac{(x-2)^2}{5} + \dfrac{(y+2)^2}{6} = 1$
Center $(2, -2)$
Major Axis along $x = 2$
Minor Axis along $y = -2$
Vertices $\left(2, -2+\sqrt{6}\right)$, $(2, -2-\sqrt{6})$
Endpoints of Minor Axis $\left(2-\sqrt{5}, -2\right)$, $\left(2+\sqrt{5}, -2\right)$
Foci $(2, -1)$, $(2, -3)$
$e = \frac{\sqrt{6}}{6}$

15. $\dfrac{(x-3)^2}{9} + \dfrac{(y-7)^2}{25} = 1$

16. $\dfrac{x^2}{39} + \dfrac{y^2}{64} = 1$

17. $\dfrac{x^2}{34} + \dfrac{y^2}{25} = 1$

18. $\dfrac{(x-8)^2}{25} + \dfrac{(y-2)^2}{4} = 1$

19. $\dfrac{(x-5)^2}{25} + \dfrac{4(y-2)^2}{75} = 1$

20. $\dfrac{(x-8)^2}{64} + \dfrac{(y+9)^2}{81} = 1$

21. Jamie and Jason should stand $100 - 25\sqrt{7} \approx 33.86$ feet from opposite ends of the gallery.

22. The arch can be modeled by the top half of $\frac{x^2}{9} + \frac{y^2}{81} = 1$. One foot in from the base of the arch corresponds to either $x = \pm 2$. Plugging in $x = \pm 2$ gives $y = \pm 3\sqrt{5}$ and since $y$ represents a height, we choose $y = 3\sqrt{5} \approx 6.71$ feet.

23. Distance from the sun to aphelion $\approx 1.0167$ AU.
Distance from the sun to perihelion $\approx 0.9833$ AU.